Dynamical reduction models: present status and future developments

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Abstract. We review the major achievements of the dynamical reduction program, showing why and how it provides a unified, consistent description of physical phenomena, from the microscopic quantum domain to the macroscopic classical one. We discuss the difficulties in generalizing the existing models in order to comprise also relativistic quantum field theories. We point out possible future lines of research, ranging from mathematical physics to phenomenology.

1. Quantum Mechanics, measurements and environment

Standard Quantum Mechanics is known to talk only about the outcomes of measurements, but it has nothing to say about the world as it is, independently of any measurement or act of observation. This is a source of serious difficulties, which have been clearly elucidated e.g. by J. Bell [1]: *It would seem that the theory is exclusively concerned about 'results of measurements', and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of 'measurer'? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little bit longer, for some better qualified system ... with a Ph.D.?*

Measuring devices, like photographic plates and bubble chambers, are very sophisticated and highly structured physical systems, which anyhow are made up of atoms; we then expect them to be ultimately described in quantum mechanical terms by means of the Schrödinger equation. What else should we do, taking into account that people are trying to describe also the entire universe quantum mechanically? But if we describe measurements in this way, then we do *not* get any outcome at the end of the process. The Schrödinger equation is linear, the superposition principle enters into play and it does it in such a way that all possible outcomes are there simultaneously in the wave function, but none of them is selected as the one which occurs physically. Yet, if we perform a measurement, we always get a definite outcome. So we have a problem with Quantum Mechanics.

In recent years, a not so new idea is gaining more and more credit: measuring devices are different from microscopic systems: they are big objects which unavoidably interact with the surrounding environment. Such an interaction turns out to be very peculiar because it destroys the coherence between different terms of a superposition and seems to reduce a pure state, where

all terms of the superposition are there simultaneously, into a statistical mixture of the states, and moreover it does so with the correct quantum probabilities. What else do we need?

This idea, that the environment somehow *naturally* guarantees the emergence of definite properties when moving from the micro to the macro, by destroying coherence among different terms of a superposition, is very appealing. But wrong. I will not spend much time on this issue, since many papers already have appeared on the subject, starting from those of Bell [2] to very recent ones [3, 4, 5]. I note here that the division between a system and its environment is not a division dictated by Nature. Such a division is arbitrarily set by the Physicist because he or she is not able to solve the Schrödinger equation for the global system; he or she then decides to select some degrees of freedom as the relevant ones, and to trace over all other degrees. This is a very legitimate division, but not compelling at all. Such a division is more or less equivalent to the division between a quantum system and a measuring device: it's artificial, just a matter of practical convenience. But if the physicist were able to analyze exactly the microscopic quantum system, the macroscopic apparatus *and* the surrounding environment together, i.e. if he or she used the Schrödinger equation to study the global system, he or she would get a very simple result: once more, because of linearity, all terms of the superposition would be present at the same time in the wave function, no one of them being singled out as that which really occurs when the measurement is performed in the laboratory.

The so called *measurement problem* of Quantum Mechanics is an open problem still waiting for a solution. Dynamical reduction models, together with Bohmian Mechanics, up to now are, in my opinion, the most serious candidates for a resolution of this problem.

2. The dynamical reduction program

Continuing quoting Bell: *If the theory is to apply to anything but highly idealized laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time, more or less everywhere? Do we not have jumping then all the time?* The basic idea behind the dynamical reduction programm is precisely this: spontaneous and random collapses of the wave function occur all the time, to all particles, whether isolated or interacting, whether they form just a tiny atom or a big measuring device. Of course, such collapses must be rare and mild for microscopic systems, in order not to disturb their quantum behavior as predicted by the Schrödinger equation. At the same time, their effect must add up in such a way that, when thousands of millions of particles are glued together to form a macroscopic system, a single collapse occurring to one of the particles affects the global system. We then have thousands of millions of such collapses acting very frequently on the macro-system, which together force its wave function to be very well localized in space.

The aim of the dynamical reduction programm is then to modify the Schrödinger evolution, by introducing new terms having the following properties:

- They must be *non-linear*, as one wants to break the superposition principle at the macroscopic level and assure the localization of the wave function of macro-objects.
- They must be *stochastic* because, when describing measurement-like situations, one needs to explain why the outcomes occur randomly; more than this, one needs to explain why they are distributed according to the Born probability rule.
- There must be an *amplification mechanism* according to which the new terms have negligible effects on the dynamics of microscopic systems but, at the same time, their effect becomes very strong for large many-particle systems such as macroscopic objects, in order to recover their classical-like behavior.

If we look carefully at these requirement, we realize that they are very mandatory: there is no assurance at all beforehand, that they can be consistently fulfilled. I think that one of the

greatest merits of the GRW proposal [6] is to have shown that they can be implemented in a consistent and satisfactory model.

3. The GRW model

Let us consider a system of N particles which, only for simplicity's sake, we take to be scalar; the GRW model is defined by the following postulates:

States. The state of the system is represented by a wave function $\psi(\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N)$ belonging to the Hilbert space $\mathcal{L}^2(\mathbf{R}^{3N})$.

Evolution. At random times, each particle experiences a sudden jump of the form:

$$\psi_t(\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N) \quad \longrightarrow \quad \frac{L_n(\mathbf{x})\psi_t(\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N)}{\|L_n(\mathbf{x})\psi_t(\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N)\|}, \tag{1}$$

where $\psi_t(\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N)$ is the statevector of the whole system at time t, immediately prior to the jump process. $L_n(\mathbf{x})$ is a linear operator which is conventionally chosen equal to:

$$L_n(\mathbf{x}) \quad = \quad \sqrt[4]{\left(\frac{\alpha}{\pi}\right)^3} \exp\left[-\frac{\alpha}{2}(\mathbf{q}_n - \mathbf{x})^2\right], \tag{2}$$

where α is a new parameter of the model which sets the the width of the localization process, and $\mathbf{q}_n$ is the position operator associated to the n-th particle; the random variable $\mathbf{x}$ corresponds to the place where the jump occurs. Between two consecutive jumps, the statevector evolves according to the standard Schrödinger equation.
The probability density for a jump taking place at the position $\mathbf{x}$ for the n-th particle is given by:

$$p_n(\mathbf{x}) \quad \equiv \quad \|L_n(\mathbf{x})\psi_t(\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N)\|^2, \tag{3}$$

and the probability densities for the different particles are independent.
Finally, it is assumed that the jumps are distributed in time like a Poissonian process with frequency λ, which is the second new parameter of the model.
The standard numerical values for α and λ are:

$$\lambda \simeq 10^{-16}\,\mathrm{sec}^{-1}, \qquad \alpha \simeq 10^{10}\,\mathrm{cm}^{-2}. \tag{4}$$

Ontology. Let the m_n be the mass associated to the n-th "particle" of the system (I should say: to what is called "a particle", according to the standard terminology); then the function:

$$\rho_t^{(n)}(\mathbf{x}_n) \equiv m_n \int d^3x_1 \dots d^3x_{n-1} d^2x_{n+1} \dots d^3x_N \, |\psi_t(\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N)|^2 \tag{5}$$

represents the *density of mass* [7] of that "particle" in space, at time t.

These are the axioms of the GRW model: as we see, words such as 'measurement', 'observation', 'macroscopic', 'environment' do not appear. There is only a *universal* dynamics governing all physical processes, and an ontology which tells how the physical world is, according to the model, independently of any act of observation.

The GRW model, together with other dynamical reduction models which have appeared in the literature, has been extensively studied (see [8] and [9] for a review on this topic); in particular—with the numerical choice for λ and α given in (4)—the following three important properties have been proved, which we will state in more quantitative terms in the following section:

- At the microscopic level, quantum systems behave almost exactly as predicted by standard Quantum Mechanics, the differences being so tiny that they can hardly be detected with present-day technology.
- At the macroscopic level, wave functions of macro-objects are almost always very well localized in space, so well localized that their centers of mass behave, for all practical purposes, like point-particles moving according to Newton's laws.
- In a measurement-like situation, e.g. of the von Neumann type, GRW reproduces—as a consequence of the modified dynamics—both the Born probability rule and the postulate of wave-packet reduction.

Accordingly, models of spontaneous wave function collapse provide a unified description of all physical phenomena, at least at the non-relativistic level, and a consistent solution to the measurement problem of Quantum Mechanics.

It may be helpful to stress some points about the world-view provided by the GRW model. According to the interpretation given by the third axiom, there are no particles at all in the theory! There are only distributions of masses which, at the microscopic level, are in general quite spread out. An electron, for example, is not a point following a trajectory—as it would be in Bohmian Mechanics—but a wavy system diffusing in space. When, in a double-slit experiment, it goes through the apertures, it literarily goes through both of them, as a classical water-wave would do. The peculiarity of the electron, which qualifies it as a quantum system, is that when we try to localize it in space by letting it interacting with a measuring device, e.g. a photographic plate, then, according to the second axiom and because of the interaction with the plate, its wave function very rapidly shrinks in space till is gets localized to a spot, the spot where the plate is impressed and which represents the outcome of the measurement. Such a behavior is not postulated *ad hoc* as done in standard Quantum Mechanics; it is a direct consequence of the universal dynamics of the GRW model.

Also macroscopic objects are waves; their centers of mass are not mathematical points, rather they are represented by some function defined throughout space. But macro-objects have a nice property: according to the GRW dynamics, each of them is always almost perfectly located in space, which means that the wave functions associated to their centers of mass are appreciably different from zero only within a very tiny region of space (whose linear extension is of order 10^{-14} m or smaller, as we shall see), so tiny that they can be considered point-like for all practical purposes. This is the reason why Newton's mechanics of point particles is such a satisfactory theory for macroscopic classical systems.

Even though the GRW model contains no particles at all, we will keep referring to micro-system as 'particles', just for a matter of convenience.

4. Dynamical reduction models and stochastic differential equations

The second axiom of the GRW model concerning the evolution of physical systems can be written more succinctly in terms of stochastic differential equations. According to the QMUPL model first proposed in [10] and subsequently studied in [11] (see also references therein), a wave function $\psi_t(\{x\}) \equiv \psi_t(x_1, x_2, \ldots x_N)$ evolves according to the following stochastic differential equation, where for simplicity we assume the dynamics to take place only in one dimension:

$$d\,\psi_t(\{x\}) \;=\; \left[-\frac{i}{\hbar}\,H_{\rm TOT}\,dt + \sum_{n=1}^{N}\sqrt{\lambda_n}\,(q_n - \langle q_n\rangle_t)\,dW_t^n - \frac{1}{2}\,\sum_{n=1}^{N}\lambda_n(q_n - \langle q_n\rangle_t)^2 dt\right]\psi_t(\{x\}); \tag{6}$$

$H_{\rm TOT}$ is the standard quantum Hamiltonian of the composite system; the symbol $\langle q_n\rangle_t$ represents the quantum average $\langle\psi_t|q_n|\psi_t\rangle$ of the position operator q_n; the random processes W_t^n ($n =$

Table 1. Decoherence rates (in $\mathrm{cm}^{-2}\mathrm{sec}^{-1}$) for different kinds of scattering processes (taken from Joos and Zeh [13]). In the last line: λ_n (in $\mathrm{cm}^{-2}\mathrm{sec}^{-1}$) as defined in (7).

Cause of decoherence	10^{-3} cm dust particle	10^{-6} cm large molecule
Air molecules	10^{36}	10^{30}
Laboratory vacuum	10^{23}	10^{17}
Sunlight on earth	10^{21}	10^{13}
300K photons	10^{19}	10^{6}
Cosmic background rad.	10^{6}	10^{-12}
COLLAPSE	10^{7}	10^{-2}

$1, \dots N$) are N independent standard Wiener processes defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$, and the coupling constants λ_n are defined as follows:

$$\lambda_n \equiv \frac{m_n}{m_0}\,\lambda_0, \tag{7}$$

where m_n is the mass of the n-th particle, while m_0 is a reference mass which we assume equal to the mass of a nucleon: $m_0 \simeq 1.7 \times 10^{-27}$ Kg. In order for the QMUPL model to be empirically equivalent to the GRW model, one has to choose $\lambda_0 \simeq 10^{-2}$ m^{-2} sec^{-1}.

The above equation has been studied quite in detail in the literature; the behavior of microscopic systems and macroscopic objects, and in particular of measurement-like situations, is the following.

4.1. Microscopic systems.
According to the dynamical reduction program, microscopic quantum systems have an existence on their own, independently of any act of observation. Anyway, they cannot be seen directly, and in order to discover their properties they have to be subjected to measurements.

As shown in [12], measurable quantities are given by averages of the form $\mathbf{E}[\langle O\rangle_t]$, where O is (in principle) any self–adjoint operator and $\mathbf{E}[\dots]$ denotes the stochastic average. It is not difficult to prove that $\mathbf{E}[\langle O\rangle_t] = \mathrm{Tr}[O\rho_t]$ where the statistical operator $\rho_t \equiv \mathbf{E}[|\psi_t\rangle\langle\psi_t|]$ satisfies the Lindblad–type equation:

$$\frac{d}{dt}\,\rho_t \;=\; -\frac{i}{\hbar}\,[H, \rho_t] - \frac{1}{2}\sum_{n=1}^{N} \lambda_n\,[q_n, [q_n, \rho_t]]\,. \tag{8}$$

This is the master equation first introduced by Joos and Zeh [13] to describe the interaction between quantum particles with a surrounding environment; consequently, only as far as experimental results are concerned, the model behaves as if the system were an open quantum system, even though in our case an environment need *not* be present for the collapses to occur. Given this, an easy way to understand the magnitude of the physical effects of the reduction process is to compare the strength of the collapse mechanism (measured by the constants λ_n) with the loss of coherence due to the presence of an environment.

Such a comparison is given in Table 1, when the system under study is a very small particle like an electron, or an almost macroscopic object like a dust particle. We see that, for most sources of decoherence, the experimentally testable effects of the collapse mechanism are weaker

than the disturbances produced by the interaction of the system with a surrounding environment. This implies that, in order to test the GRW effects, one has to keep a quantum system isolated for a sufficiently long time, from most sources of decoherence, and this is difficult to achieve, unless very sophisticated experiments are performed (more about this in the following). The analysis then shows that the predictions of the GRW model are in good agreement with standard quantum mechanical predictions.

4.2. Macroscopic objects

Let us now consider what happens not to a small quantum system, but to a macroscopic object. For the purposes of our analysis, it is convenient to switch to the center–of–mass (R) and relative ($\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_N$) coordinates:

$$R \;=\; \frac{1}{M}\sum_{n=1}^{N} m_n\, x_n \qquad x_n \;=\; R + \tilde{x}_n, \qquad\qquad M \;=\; \sum_{n=1}^{N} m_n; \tag{9}$$

let Q be the position operator for the center of mass and $\tilde{q}_n$ ($n = 1 \ldots N$) the position operators associated to the relative coordinates. It is not difficult to show that, under the assumption $H_{\rm TOT} = H_{\rm CM} + H_{\rm rel}$, the dynamics for the center of mass and that for the relative motion decouple; in other words, $\psi_t(\{x\}) = \psi_t^{\rm CM}(R) \otimes \psi_t^{\rm rel}(\{\tilde{x}\})$ solves Eq. (6) whenever $\psi_t^{\rm CM}(R)$ and $\psi_t^{\rm rel}(\{\tilde{x}\})$ satisfy the following equations:

$$d\psi_t^{\rm rel}(\{\tilde{x}\}) \;=\; \left[-\frac{i}{\hbar}\,H_{\rm rel}\,dt + \sum_{n=1}^{N}\sqrt{\lambda_n}\,(\tilde{q}_n - \langle\tilde{q}_n\rangle_t)dW_t^n - \frac{1}{2}\sum_{n=1}^{N}\lambda_n(\tilde{q}_n - \langle\tilde{q}_n\rangle_t)^2 dt\right]\psi_t^{\rm rel}(\{\tilde{x}\}), \tag{10}$$

$$d\psi_t^{\rm CM}(R) \;=\; \left[-\frac{i}{\hbar}\,H_{\rm CM}\,dt + \sqrt{\lambda_{\rm CM}}\,(Q - \langle Q\rangle_t)dW_t - \frac{\lambda_{\rm CM}}{2}\,(Q - \langle Q\rangle_t)^2 dt\right]\psi_t^{\rm CM}(R), \tag{11}$$

with:

$$\lambda_{\rm CM} \;=\; \sum_{n=1}^{N}\lambda_n \;=\; \frac{M}{m_0}\,\lambda_0. \tag{12}$$

The first of the above equations describes the internal motion of the system: it basically tells that, since the constants λ_n are very small in magnitude, the internal structure is described in agreement with the standard Schrödinger equation, modulo small deviations of the type discussed in the previous subsection. We now focus our attention on the second equation.

Eq. (11) shows that the reducing terms associated to the center of mass of a composite system are equal to those associated to a particle having mass equal to the total mass M of the whole system. The constant $\lambda_{\rm CM}$ has now a much larger value than that of the λ_n, thus we expect the dynamics of the center of mass to be completely different from that of the microscopic quantum particles discussed in the previous section. This is precisely the *amplification mechanism* we talked about before: tiny collapses associated to each particle sum up and produce a very strong collapse on the global system.

As a matter of fact, in Ref. [11] it has been proven that, for macroscopic values of M, an initially spread wave function is very rapidly localized in space, within a time interval much smaller than the perception time of a human observer, and it reaches asymptotically the value (for an isolated system)

$$\sigma_q(\infty) \;\simeq\; \left(1.5\times 10^{-15}\sqrt{\frac{\rm Kg}{M}}\right)\,{\rm m} \;\simeq\; \begin{cases} 4.6\times 10^{-14}\;\;{\rm m} & \text{for an 1–g object,} \\ 5.9\times 10^{-28}\;\;{\rm m} & \text{for the Earth.} \end{cases} \tag{13}$$

As we see, the asymptotic spread of the wave function of the center of mass of a macroscopic object is very very small, so small that the wave function can be considered, for all practical purposes like a point in space! A similar localization occurs also in momentum space, within the limits allowed by Hesienberg's uncertainty principle. This is how dynamical reduction models justify the point-like behavior of macroscopic classical particles.

But particles move in space: do they move according to Newton's laws? It is easy to show that the average value $\mathbf{E}[\langle Q\rangle_t]$ of the mean position and $\mathbf{E}[\langle P\rangle_t]$ of the mean momentum satisfy the following equations:

$$\frac{d}{dt}\,\mathbf{E}\,[\langle Q\rangle_t] \;=\; \frac{i}{\hbar}\mathbf{E}\,[\langle [H_{\rm CM}, Q]\rangle_t]\,, \qquad\qquad \frac{d}{dt}\,\mathbf{E}\,[\langle P\rangle_t] \;=\; \frac{i}{\hbar}\mathbf{E}\,[\langle [H_{\rm CM}, P]\rangle_t]\,, \tag{14}$$

which can be considered as the stochastic extension of Ehrenfest's theorem; we then recover the classical equation of motion, in the appropriate limit. But this is not enough: the above equations refer only to average values, while we want the motion to be approximately Newtonian for single realizations of the stochastic process, otherwise the model would not reproduce classical mechanics at the macro-level. In Ref. [11] it has been proven, for Gaussian solutions and in the case of an isolated system, that the variance of $\langle Q\rangle_t$ associated to the motion of the center of mass evolves as follows:

$$\mathbf{V}[\langle Q\rangle_t] \;\simeq\; \begin{cases} (1.1\times 10^{-31}\, t/\text{sec})\ \text{m}^2 & \text{for a } 1g \text{ object,} \\ \\ (1.8\times 10^{-59}\, t/\text{sec})\ \text{m}^2 & \text{for the Earth,} \end{cases} \tag{15}$$

for $t < 2.0\times 10^4$ sec, while for longer times it increases like t^3. We see that for a macro–object and for very long times (much longer that the time during which a system can be kept isolated) the fluctuations are so small that, for all practical purposes, they can be safely neglected; this is how *classical determinism* is recovered within our stochastic model. Note thus that, contrary to the behavior of the reduction mechanism, which is amplified when moving from the micro- to the macro-level, the fluctuations associated to the motion of microscopic particles interfere destructively with each other, in such a way that the diffusion process associated to the center of mass of an N-particle system is much weaker than that of the single components.

The above results imply that the actual values of $\langle Q\rangle_t$ (and also of $\langle P\rangle_t$) are practically equivalent to their stochastic averages, which obey Eqs. (14); we than have that $\langle Q\rangle_t$ and $\langle P\rangle_t$ practically evolve according to the classical laws of motion (in the appropriate physical situations) for most realizations of the stochastic process. Since, for very localized states like those having a spread given by Eq. (13), $\langle Q\rangle_t$ represents the spot where the wave function is concentrated, we reach the following conclusion: in the macroscopic regime, the wave function of a macroscopic system behaves, for all practical purposes, like a point–like particle moving deterministically according to Newton's laws of motion.

4.3. Measuring situation

In a recent paper [14] we have analyzed the evolution of the wave function as predicted by Eq. (6), when a macroscopic system acting as a measuring device interacts with a microscopic quantum system in such a way to measure one of its properties. The paper contains a mathematical analysis of the situation, and proves, also giving precise estimates, the following results:

(i) whichever the initial state of the microscopic system, throughout the entire measurement process the center of mass of the pointer is always extremely well localized and moves as expected, from the ready state position to its final position.

(ii) the only possible outcomes correspond to those given by standard quantum mechanics, with probability infinitesimally close to 1;

(iii) the probability of getting a certain outcome is given by the *Born probability rule* within an exceedingly high degree of approximation;

(iv) after the measurement, the microscopic system is in a state which practically coincides with an eigenstate of the observable which has been measured, corresponding to the eigenvalue which has been observed.

This proves rigorously what was the first goal of the original GRW model: to provide a consistent solution to the measurement problem of Quantum Mechanics.

4.4. Identical particles

The GRW model, as well as the QMUPL model previously discussed, refers to a non-relativistic system with an arbitrary number of *distinguishable* particles; the model has been successfully generalized to include also *identical* particles. The best known example is the CSL model [15] which is based on the following stochastic differential equation:

$$d\psi_t = \left[-\frac{i}{\hbar}\,H\,dt + \sqrt{\gamma}\int d^3x\,(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t)\,dW_t(\mathbf{x}) - \frac{\gamma}{2}\int d^3x\,(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle_t)^2\,dt\right]\psi_t, \tag{16}$$

where the symbol $\langle N(\mathbf{x})\rangle_t$ denotes the quantum average of the operator $N(\mathbf{x})$, which is an *average density number* operator defined as follows:

$$N(\mathbf{x}) \;=\; \left(\frac{\alpha}{2\pi}\right)^{3/2}\sum_s \int d^3y\, e^{-\frac{\alpha}{2}(\mathbf{x}-\mathbf{y})^2}\, a^\dagger(s,\mathbf{y})\, a(s,\mathbf{y}), \tag{17}$$

where $a^\dagger(s,\mathbf{y})$ ($a(s,\mathbf{y})$) is the creation (annihilation) operator of a particle of spin s at position $\mathbf{y}$ of space. Note that, instead of having different Wiener processes attached to each particle as in Eq. (6), which make the particles follow different histories and thus be distinguishable, we now have a continuum of independent Wiener processes $W_t(\mathbf{x})$ (one for each point of space) which are not attached to any particular particle, but only to their (average) number density; hence the evolution respects the symmetry or anti-symmetry properties of the wave function. The constant γ has been set equal to $\lambda(4\pi/\alpha)^{3/2}$, where λ is given in (4), in order for GRW and CSL to coincide for one particle.

The CSL model has been widely studied in the literature, and we refer the reader to [8] for the details. See [16] for a discrete, GRW-like, reduction model for identical particles.

5. Relativistic dynamical reduction models

The great challenge of the dynamical reduction program is to formulate a consistent model of spontaneous wave function collapse for relativistic quantum field theories; many attempts have been proposed so far, none of which is as satisfactory as the non-relativistic GRW model.

The first attempt [17] aimed at making the CSL model relativistically invariant by replacing Eq. (16) with a Tomonaga-Schwinger equation of the type:

$$\frac{\delta\psi(\sigma)}{\delta\sigma(x)} = \left[-\frac{i}{\hbar}\,\mathcal{H}(x) + \sqrt{\gamma}\,(\mathcal{L}(x) - \langle\mathcal{L}(x)\rangle)\,V(x) - \frac{\gamma}{2}\,(\mathcal{L}(x) - \langle\mathcal{L}(x)\rangle)^2\right]\psi(\sigma), \tag{18}$$

where now the wave function is defined on an arbitrary space-like hypersurface σ of space-time. The operator $\mathcal{H}(x)$ is the Hamiltonian density of the system (x now denotes a point in space-time), and $\mathcal{L}(x)$ is a *local* density of the fields, on whose eigenmanifolds one decides to localize the wave function. The c-number function $V(x)$ is a stochastic process on space-time with mean equal to zero, while the correlation function—in order for the theory to be Lorentz invariant in the appropriate stochastic sense [17]—must be a Lorentz scalar. And here the problems arise!

The simplest Lorentz invariant choice for the correlation function is:

$$\mathbf{E}[V(x)V(y)] = \delta^{(4)}(x-y), \tag{19}$$

which however is not physically acceptable as it causes an infinite production of energy per unit time and unit volume. The reason is that in Eq. (18) the fields are *locally* coupled to the noise which, when it is assumed to be *white*, is too violent, so to speak, and causes too many particles to come out of the vacuum. To better understand the situation, let us go back to the non-relativistic Eq. (16): also there we basically have a white-noise process, which however is not coupled locally to the quantum field $a^\dagger(s,\mathbf{y})a(s,\mathbf{y})$, the coupling being mediated by the smearing Gaussian function appearing in the definition of $N(\mathbf{x})$. One can compute the energy increase due to the collapse mechanism, which turns out to be *proportional* to α. Now, if we want to have a local coupling between the quantum field and the noise, we must set $\alpha \to +\infty$, in which case the energy automatically diverges to infinity.

The simplest way out one would think of, in order to cure this problem of Eq. (18), is to replace the local coupling between the noise and the quantum field by a non-local one, as in the CLS equation (16); this procedure would essentially amount to replacing the white noise field with a non-white one. In both cases we need to find a Lorentz invariant function which either smears out the coupling or replaces the Dirac-delta in the definition of the correlation function (19). This however is not a straightforward task, for the following reason.

One of the reasons why the third term $(\gamma/2)\left(\mathcal{L}(x) - \langle\mathcal{L}(x)\rangle\right)^2$ appears in Eq. (18) is to guarantee that the collapse mechanism occurs with the correct quantum probabilities (for those experts in stochastic processes, the third term is such that the equation embodies an appropriate martingale structure); if we change the noise, we then have to change also the third term, and it turns out that we have to replace it with a *non-local* function of the fields [18, 19]. But, having a non-local function of the fields jeopardizes the entire (somehow formal) construction of the theory based on the Tomanaga-Schwinger equation, as the integrability conditions are not automatically satisfied, and it is very likely that the model will turn out to be inconsistent.

What we have briefly described is the major obstacle to finding a relativistic dynamical reduction model. We want to briefly mention three research programs which try to overcome such an impasse.

P. Pearle has spent many years in trying to avoid the infinite energy increase of relativistic spontaneous collapse models, e.g. by considering a tachyonic noise in place of a white noise as the agent of the collapse process [20], obtaining suggestive results. Unfortunately, as he has recently admitted [19], the program so far did not succeed.

Dowker and Henson have proposed a spontaneous collapse model for a quantum field theory defined on a 1+1 null lattice [21, 22], studying issues like the non-locality of the model and the no-faster-than-light constraint. More work needs to be done in trying to apply it to more realistic field theories; in particular, it would be important to understand if, in the continuum limit, one can get rid of the divergences which plague the relativistic CSL model.

In a recent paper [23], generalizing a previous idea of Bell [24], Tumulka has proposed a discrete, GRW-like, relativistic model, for a system of N non-interacting particles, based on the multi-time formalism with N Dirac equations, one per particle; the model fulfills all the necessary requirements, thus it represents a promising step forward in the search for a relativistic theory of dynamical reduction. Now it is important to understand whether it can be generalized in order to include also interactions.

It is rather discomforting that, after so many years and so many efforts, no satisfactory model of spontaneous wave function collapse for relativistic quantum field theories exists. And some have started to wonder whether there is some fundamental incompatibility between the dynamical reduction program and relativity. In this regard, we mention the analysis of Ref. [25], where a toy model of spontaneous wave function collapse is analyzed: the collapse mechanism

is supposed to occur instantaneously along all spacelike hypersurfaces crossing the center of the jump process; in spite of this superluminal effect, the whole picture is perfectly Lorentz invariant, it agrees with quantum mechanical predictions, it does not lead to any contradiction, e.g. it does not allow faster-than-light signalling and, moreover, different inertial observers always agree on the outcomes of experiments. Unfortunately, the missing piece (which would make the toy model a real physical model) is the dynamics for the reduction mechanism; in any case, this model suggests that there is no reason of principle forbidding the relativistic reduction program.

6. Open questions and future developments

Apart the important issue of finding satisfactory relativistic dynamical reduction models, there is still much work to be done and many open questions to be answered, at different levels, ranging from mathematical to experimental physics. We conclude this paper with a (partial) list some of interesting open problems.

6.1. Open questions: Mathematical Physics

Let us consider once more Eq. (6), which for a single particle reads:

$$d\,\psi_t(x) \;=\; \left[-\frac{i}{\hbar}\,H\,dt + \sqrt{\lambda}\,(q-\langle q\rangle_t)\,dW_t - \frac{\lambda}{2}\,(q-\langle q\rangle_t)^2 dt\right]\psi_t(x); \tag{20}$$

this is the simplest known continuous generalization of the original GRW model, and existence and uniqueness of solutions have been proved already in a number of theorems [26, 27]; still, many important properties have not been studied yet. For example: is it possible to write down explicitly the general solution of Eq. (20) for the three most significant, usually exactly solvable, physical systems, namely the free particle ($H = p^2/2m$), the harmonic oscillator ($H = p^2/2m + m\omega^2/2$) and the hydrogen atom ($H = p^2/2m - e^2/r$)? What about the other types of Hamiltonian operator which can be solved analytically in the standard quantum case?

Another important type of problems concerns the large time behavior of the solution of Eq. (20). In the *free-particle* case, various authors in different ways proved or were close to proving, with a different degree of rigor, that any initial state belonging to the domain of the equation, with the possible exception of a subset of measure zero, converges to a Gaussian wave function with a fixed spread both in position and momentum, while the respective mean values diffuse. Apart from the free particle, for which class of Hamiltonian operators H does almost any initial state converge to some fixed state with a finite spread in position and momentum? Does this class contain the physically most significant Hamiltonian operators? With what rate does an initial state converge to the asymptotic state? Does the amplification mechanism work as expected, i.e. in such a way that the bigger the particle, the faster the collapse?

6.2. Open questions: Theoretical Physics

I think that many researchers in this field consider the dynamical reduction program as a first important step towards a formulation of a new theory for microscopic physical processes, which supersedes quantum mechanics; the big question is: what does this theory look like? There has been a lot of speculation in this regard, which dates back to Einstein who thought of quantum mechanics as a provisional theory which eventually will end up in being a statistical approximation of a deeper theory; as far as I know the only concrete proposal along these lines is the one put forward by S. Adler in his recent book [28]. He assumes precisely that quantum mechanics is not a fundamental theory of nature but an emergent phenomenon arising from the statistical mechanics of matrix models with a global unitary invariance. The book is entirely devoted to showing how that idea can be implemented within a concrete (and highly sophisticated) mathematical framework, and we invite the reader to look at it for all necessary details.

Table 2. The table shows the *upper bounds* on the possible numerical value of the collapse parameter λ_0, set by present-day observational data. The values are taken from [33]. E.g. for fullerene experiments, the number $5 \times 10^{12}\lambda_0$ means that the upper bound is 5×10^{12} times larger than the standard value λ_0 as given in Sec. 4.

Fullerene diffraction	Decay of super-currents	Radiation by free electrons	11 KeV photons from Ge	Proton decay
$5 \times 10^{12}\lambda_0$	$10^{14}\lambda_0$	$10^{12}\lambda_0$	$3 \times 10^{14}\lambda_0$	$10^{18}\lambda_0$
Hydrogen dissociation	Heating of protons	I.G.M.	Inter-stellar dust grains	
$4 \times 10^{17}\lambda_0$	$10^{12}\lambda_0$	$10^{8\pm1}\lambda_0$	$10^{15}\lambda_0$	

Another important issue, related to the previous one, is the following. In all dynamical reduction models so far developed, the stochastic process responsible for the collapse of the wave function is a sort of mathematical entity without an existence on its own, whose only job is to localize the wave function. I find it very tempting to imagine that this field is *real*, that it has its own equations of motion and acts on the quantum system, but also that the quantum system acts back on it. It is also tempting to say that this field is not a new field of nature, but the only field that has not been successfully quantized yet, i.e. the gravitational field. The research has already moved in this direction [9, 29, 30, 31], and it is very exciting and worthwhile pursuing it. It could also clarify a rather delicate issue connected with the violation of the energy conservation principle in dynamical reduction models.

As discussed in [11], the collapse mechanism induces a sort of diffusion process on the wave function in momentum space, which makes it pick up higher and higher components in momentum, which in turn show up as an increase of the energy of the system. With the choice made in (4) for the parameters λ and α, such a violation is very tiny and hardly detectable with present day technology; still, it is present and some people find it disturbing. Now, if the stochastic field has to be regarded as a real physical field with a reality on its own and its own equations of motion, it seems natural to think that we are making a mistake in assuming that the energy of a quantum system should be conserved; in the calculations we should instead consider the energy of the stochastic field *together* with the energy of the quantum system, as the global energy which should be conserved. In this way, there is serious hope to restore the principle of energy conservation within dynamical reduction models. A first promising step in this direction has been put forward in Ref. [32].

6.3. Open questions: Phenomenology and Experiments

Dynamical reduction models, by modifying the Schrödinger equation, are predictively different from standard quantum mechanics; it becomes then interesting to look for situations where it would be easier to test these models against the standard quantum theory. Indeed, the importance of such a research goes far beyond the dynamical reduction program itself, as it ultimately would aim at testing one of the most characteristic traits of quantum mechanics, namely the superposition principle.

In a recent paper [33], S. Adler has done an exhaustive and up-to-date review of the most plausible scenarios where it is more likely to detect possible violations of the superposition principle, and the presence of a spontaneous collapse mechanism. These results are summarized in table 2.

Basically, there are two scenarios where it is more convenient to look for possible GRW-effects: high precision experiments on micro-systems, and cosmological data; the first are reported on the upper row, and the second on the lower row. As we see, for the standard values given in Eq. (4), the constraints are rather weak and there is no hope that in the near future such effects can be possibly tested.

However, in [33] Adler notices that, given (4), a wave function is not reduced when a latent image is formed, in photography or etched track detection. Since one would think it very natural to assume the localization process to occur already at the stage of latent image formation (as a latent image can be safely stored for very long times and only afterwards developed or etched), Adler suggests to increase the standard numerical value of λ_0 by order of $2 \times 10^{9\pm2}$, to guarantee the collapse to occur already at this stage. This is a very suggestive hypothesis, since it implies that in the very near future technology will be available, which will allow for a test of dynamical reduction models.

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